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METEOROLOGY AND AIR QUALITY

Using Dimension Analysis in Search of a Stable Boundary Layer Height Formulation

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Background

The boundary layer height is a key issue for turbulent diffusion calculations in weather prediction models and for pollutant dispersion calculations in air quality models.

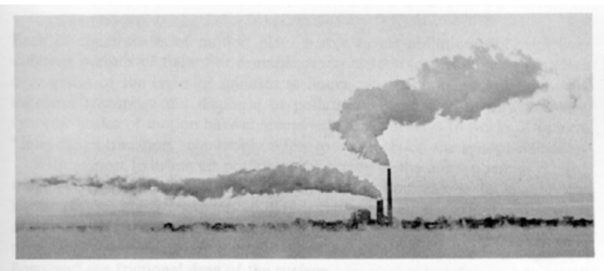

Figure 1: Pollutant dispersion above and below the boundary layer height

Zilitinkevich and Mironov (1996) propose a multi-limit equation for the nighttime h that includes Earth's rotation (f), surface based turbulence (u_*), surface buoyancy flux (B_s) and background stratification (N):

$$\left(\frac{hf}{C_n u_*}\right)^2 + \frac{h}{C_s L} + \frac{Nh}{C_i u_*} + \frac{h\sqrt{|fB_s|}}{C_{sR} u_*^2} + \frac{h\sqrt{|fN|}}{C_{iR} u_*} = 1 \qquad (1)$$

The main drawback of this formula is the amount of coefficients to be obtained from observations. In addition Eq (1) gives poor prediction for CASES99 data (Fig 2).

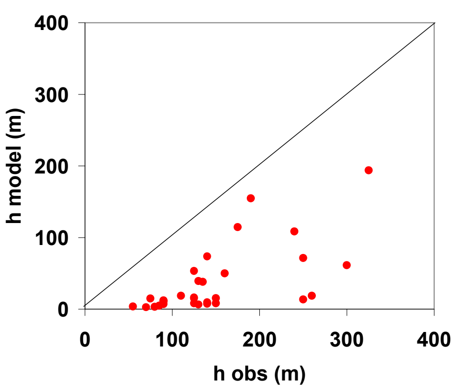

Figure 2: Verification of Eq. (1) with CASES-99 observations

Dimension Analysis

From a fundamental point of view, Eq. (1) violates the laws of dimensional analysis, since the provided relevant quantities are f, B_s, u_*, N and h, have 2 basic dimensions. Consequently, only 3 dimensionless groups are allowed, instead of the 5 used. With dimensional analysis we find for the 3 groups (L is the Obukhov length):

$$\Pi_1 = \frac{h}{L} \qquad \Pi_2 = \frac{N}{f} \qquad \Pi_3 = \frac{B_s}{f u_* N h}$$

The functional form between these 3 groups is determined from WINTEX observations as analysed by Joffre et al. (2001). Figure 3 shows the dependence of Π_1 versus Π_3 for classes of Π_2. The data order in classes of N/f.

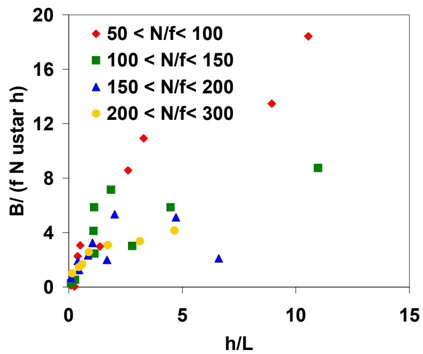

Figure 3: Relationship between the 3 relevant groups for WINTEX observations

Alternative Formulation

From the dimensionless groups we obtain a new formula for the stable boundary layer height h:

$$h = L\left(\frac{B_s}{3 N u_* f L}\right)^{\frac{1}{1.8-0.001\frac{N}{f}}} \qquad (2)$$

We verify Eq. (2) for two different sites that differ in roughness, latitude and land use: Cabauw (52°N, z_0 = 0.20 m, grass) and CASES-99 (37°N, z_0 =0.03 m, prairie) (Fig 3). Furthermore Eq.(2) gives good agreement with Large Eddy Simulations from the GABLS intercomparison.

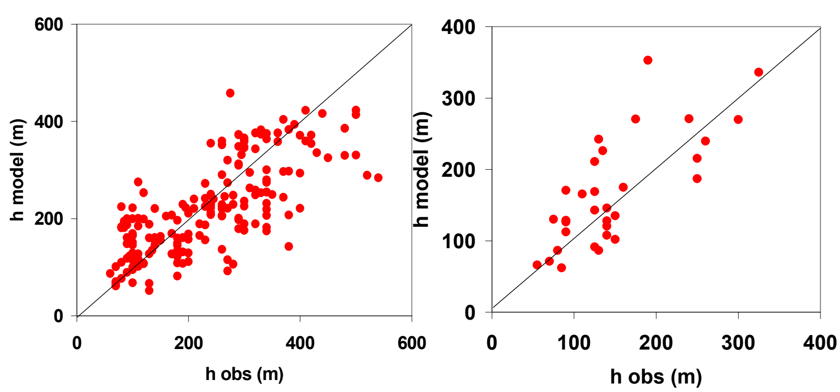

Figure 4: Verification of Eq. (2) against Cabauw (left) and CASES-99 (right) observations

Conclusion

We derived an alternative formulation for the height of the nighttime boundary layer, which is robust compared to observations and LES model results

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References:

Zilitinkevich, S.S. and D.V. Mironov, 1996: *BLM*, **81**, 325-351.
Joffre, S.M., et al., 2001, *BLM*, **99**, 429-450.

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